



# 2021 US NATIONAL ASTRONOMY OLYMPIAD FINAL QUESTION SET



Rice University Space Institute Space Science Outreach

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This test is designed to be taken with an answer sheet to record multiple-choice, short-answer, and free-response solutions from the student. This printable answer sheet has been provided along with the exams. Any material written directly on this examination document will not be scored. Students are to mark all selections on the answer sheet, not the exam.

No strict time limit is provided to complete all free response and multiple choice questions. A calculator may be used. The full exam is 25 questions long, consisting of eighteen multiple-choice, five short-answer, and two multi-part free-response questions.

Record all solutions on the answer sheet provided. There is only one correct or best answer to each question. For multiple-choice questions, when you select your choice, blacken the corresponding space on the answer sheet with your pencil. Make a heavy full mark, but no stray marks. If you decide to change your answer, be certain to erase your original mark completely.

For short-answer and free-response questions, please write your response inside the designated area for the corresponding question in the answer sheet. Should this space be insufficient, an additional page with space for an unspecified question is provided. Please print and append as many copies of this additional page as is necessary to complete your response.

Please read and sign the following agreement:

*On penalty of disqualification and removal from the USNAO, I pledge that I have received no unauthorized aid (including, but not limited to: external academic resources and assistance from others) on this examination.*

\_\_\_\_\_  
Printed Full Name

\_\_\_\_\_  
Date

\_\_\_\_\_  
Signature

**1. A sidereal day is 3 minutes and 56 seconds shorter than a solar day. This time difference is due to the**

- (A) Earth's rotation around its axis.
- (B) Sun's rotation around its axis.
- (C) Sun's motion around the galactic center.
- (D) Earth's revolution around the Sun.

**2. Polaris will not always be the North star due to**

- (A) the sidereal day being shorter than the solar day.
- (B) the precession of the Earth's axis.
- (C) the Earth's period being slightly longer than 365 days.
- (D) the Solar wind blowing the Earth away from the sun.

**3. An observer standing  $30^\circ$  north from the equator observes a star at zenith. If the observer moves along the same longitude  $90^\circ$  to the south from where he or she was, what would be the new altitude of the star?**

- (A)  $0^\circ$
- (B)  $30^\circ$
- (C)  $60^\circ$
- (D)  $90^\circ$

**4. Redshift of spectral lines shows us the velocity that galaxies are moving away from us. The**

**recession speed is given by  $v = \frac{d\lambda}{\lambda} c$ . The**

**observed wavelength of a galaxy's H $\alpha$  absorption line is 6564.64 Å. If the laboratory wavelength ( $\lambda$ ) of H $\alpha$  is 6563 Å and the speed of light ( $c$ ) is 300,000 km·s $^{-1}$ , find the recession speed of the galaxy.**

- (A) 35 km·s $^{-1}$
- (B) 50 km·s $^{-1}$
- (C) 65 km·s $^{-1}$
- (D) 75 km·s $^{-1}$

**5. A photon's energy is equal to Planck's constant times its frequency. Find the energy of a photon with a wavelength of 5000 Å ( $5 \times 10^{-7}$  m). The speed of light is  $3 \times 10^8$  m·s $^{-1}$  and Planck's constant is  $6.625 \times 10^{-34}$  J·s. ( $E = hf$ )**

- (A)  $39.75 \times 10^{38}$  J
- (B)  $39.75 \times 10^{-20}$  J
- (C)  $39.75 \times 10^{-20}$  J·s
- (D)  $30.34 \times 10^{30}$  J·s

**6. The hydrogen in the interstellar medium is generally quite cold (if it is not in the vicinity of warm stars) and found in its atomic or neutral ground state. Such clouds are called HI regions. Cold interstellar hydrogen emits radiation in radio region at a wavelength of**

- (A) 26 cm
- (B) 21 cm
- (C) 5910 Å
- (D) 1000 nm

**7. The term "Hot Jupiter" is a commonly used word among the planetary scientists in the world. This refers to the fact that**

- (A) Jupiter formed much closer to the Sun a few billion years ago.
- (B) Volcanic eruptions heat up Jupiter's surface.
- (C) Some newly-discovered extrasolar planets are Jupiter sized but much closer to the parent star than the distance between the Sun and Jupiter.
- (D) Jupiter may initiate fusion and become a star in the future.

**8. Stars are born, evolve and die. The primary mass of the star decides the way it ends. A star which is comparable to the Sun's mass will release a**

- (A) Supernova
- (B) Nova
- (C) Planetary nebula
- (D) X-ray burst

9. A, B, and C are three stars with temperatures of 6000K, 24000K, and 3500K, respectively.

What is the possible colour sequence of A, B and C, respectively?

- (A) Yellowish, Bluish, Reddish
- (B) Reddish, Bluish, Yellowish
- (C) Bluish, Yellowish, Reddish
- (D) Yellowish, Reddish, Bluish

10. The brightest star of the Scorpio constellation is

- (A) Sirius
- (B) Regulus
- (C) Antares
- (D) Polaris

11. There are two stars A and B with apparent magnitudes of 1 and 3, respectively. The correct expression about their apparent brightness is

- (A) Star A is 6 times brighter than star B.
- (B) Star A is 6 times fainter than star B.
- (C) Star A is 6.310 times brighter than star B.
- (D) Star A is 2.512 times brighter than star B.

12. The weight of a woman on Earth's surface is 600 N. If she goes to the planet Mars, her weight on the Martian surface would be

- (A) 200 N.
- (B) 100 N.
- (C) 600 N.
- (D) 1200 N.

13. The escape velocity ( $v$ ) of an object (at its surface) depends on its mass ( $M$ ) and radius ( $R$ ), and is given by  $v = \sqrt{2GM/R}$ . (Here, the universal gravitational constant  $G = 6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ ). Suppose a star with an escape velocity of  $1800 \text{ km} \cdot \text{s}^{-1}$  subsequently becomes a neutron star having a radius 105 times smaller than the original star. The escape velocity of the neutron star would be

- (A) 200,000  $\text{km} \cdot \text{s}^{-1}$ .
- (B) 180,000  $\text{km} \cdot \text{s}^{-1}$ .
- (C) 180,000  $\text{m} \cdot \text{s}^{-1}$ .
- (D) 570,000  $\text{km} \cdot \text{s}^{-1}$ .

14. A comet orbiting around the Sun has a perihelion distance of 2.0 AU and an aphelion distance of 6.0 AU. The eccentricity ( $e$ ) of its elliptical orbit is

- (A) 0.4.
- (B) 0.5.
- (C) 0.7.
- (D) 1.0.

15. Using Kepler's third law, compute the period of an asteroid orbiting around the Sun with a semi-major axis of 4.0 AU. The period (in Earth years) is

- (A) 3.
- (B) 4.
- (C) 8.
- (D) 76.

16. If the orbital speed of earth is 30 km/s and distance between the Earth and the Sun is  $1.5 \times 10^{11} \text{ m}$ , the mass of the Sun would be (Note:  $G = 6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ )

- (A)  $1.9 \times 10^{30} \text{ kg}$ .
- (B)  $2.0 \times 10^{30} \text{ kg}$ .
- (C)  $2.1 \times 10^{30} \text{ kg}$ .
- (D) None of the above.

17. If the focal length of the objective of a telescope is 1 m and the focal length of its eyepiece is 10 mm, the magnification of the telescope is

- (A) 10.
- (B) 100.
- (C) 200.
- (D) 1000.

18. A star observed for a full year has a parallax angle of  $0.29''$ . The distance to that star in light years is

- (A) 10.
- (B) 100.
- (C) 200.
- (D) 1000.

**END OF MULTIPLE-CHOICE**

# SHORT-ANSWER

Please write your answers to the short-answer and free-response questions in the designated areas of the answer sheet. Material written on these pages will not be scored, but may be used for scratch work.

**19. Describe the Titius-Bode law, and estimate the distance to Saturn from the Sun in Astronomical Units (AU).**

**20. Draw a sketch of the celestial sphere and label the i) ecliptic, ii) celestial equator, iii) Vernal and Autumnal equinoxes. On the same diagram or a separate diagram indicate the equatorial coordinate system: Right Ascension and Declination.**

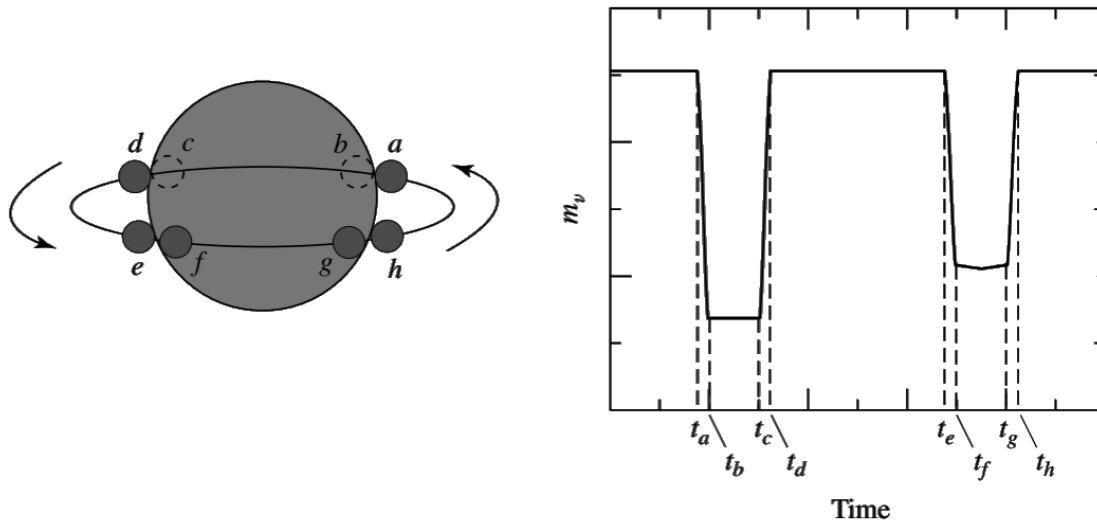
**21. If the Moon's tides pull the water away from the Earth, why are there two high tides a day?**

**22. Do the tides cause the Earth's rotation to speed up or slow down? Why?**

**23. How many solar eclipses can you have in a year? Can they all be total?**

**END OF SHORT-ANSWER**

# FREE RESPONSE Q1



**FIGURE 7.9** The light curve of an eclipsing binary for which  $i = 90^\circ$ . The times indicated on the light curve correspond to the positions of the smaller star relative to its larger companion. It is assumed in this example that the smaller star is hotter than the larger one.

Note: Free-Response Question 1 will relate to the above figure (Fig 7.9). This figure is not an exact replication of the system described in the question, but rather demonstrates the idea of “dips” in light curves produced by eclipsing binaries.

**FRQ 1.** From the light and velocity curves of an eclipsing, spectroscopic binary star system, it is determined that the orbital period is 6.31 yrs, and the maximum radial velocities of Stars A and B are 5.4 km/s and 22.4 km/s, respectively. Furthermore, the time period between first contact and minimum light ( $t_b - t_a$ ) is 0.58 days, the length of the primary minimum ( $t_c - t_b$ ) is 0.64 days, and the apparent bolometric magnitudes of maximum, primary minimum, and secondary minimum are 5.40 magnitudes, 9.20 magnitudes, and 5.44 magnitudes, respectively. From this information, and assuming circular orbits, find the:

- (A) Ratio of stellar masses.
- (B) Sum of the masses (assuming  $i = 90^\circ$ ).
- (C) Individual masses.
- (D) Individual radii.
- (E) Ratio of the effective surface temperatures of the two stars.

## FREE RESPONSE Q2

The Stefan-Boltzmann law states that the energy radiated per unit area per unit time from the surface of a blackbody is equal to  $\sigma T^4$ , where  $T$  is the blackbody's effective surface temperature in Kelvin and  $\sigma$  is the Stefan-Boltzmann constant. ( $\sigma = 5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$ ). To find the total luminosity of a blackbody, this quantity can be summed over the surface area of the object. Thus, the luminosity of the Sun can be expressed as  $L_{\odot} = 4\pi R_{\odot}^2 \cdot \sigma T_{\odot}^4$ .

In order to estimate a planet's equilibrium temperature, assume that the planet is a spherical blackbody of radius  $R_p$  and surface temperature  $T_p$  in a circular orbit a distance  $D$  from the Sun. For now, we can assume that the planet's temperature is uniform over its surface and that the planet reflects a fraction  $a$  of incoming solar energy ( $a$  is known as the planet's albedo). From the condition of thermal equilibrium, the energy emitted by the planet's blackbody radiation must be equal to the energy received from the Sun's light. It can then be shown that

$$T_p = T_{\odot} (1 - a)^{1/4} \sqrt{\frac{R_{\odot}}{D}} \quad (1)$$

**FRQ 2.** For Mercury, a slowly rotating planet with no appreciable atmosphere, Eq. (1) for a planet's surface temperature must be modified. In particular, the assumption that the temperature is uniform over the surface of the planet is no longer valid.

- (A) Assuming (incorrectly) that Mercury is in synchronous rotation about the Sun, show that the temperature at a latitude  $\theta$  north or south of the *subsolar point* (the point on the equator closest to the Sun) is approximately given by

$$T_p = (\cos \theta)^{1/4} (1 - a)^{1/4} T_{\odot} \sqrt{\frac{R_{\odot}}{D}}$$

- (B) Draw and label a graph of  $T_p$  vs  $\theta$ . Mercury's albedo is 0.06.
- (C) What is the approximate temperature of the planet at the subsolar point?
- (D) At what latitude does the temperature drop to 273 K? This is the freezing point of water at the surface of the Earth.
- (E) Would you expect to find ice on Mercury at a temperature of 273 K? Why or why not?

**END OF EXAMINATION**